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A Representation of Concepts for their Computerization[1]

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A lattice theoretical description of concept hierarchies is developed using for attributes the terms "given", "negated", "open" and "impossible" as the truth-values of a four-valued logic. Similar to the theory of B. Ganter and R. Wille (6) so does this framework permit a precise representation of the usual interdependences in a field of related concepts – such as superconcept, subconcept, contrary concepts etc. –, whenever the concepts under consideration can be sufficiently described by the presence or absence of certain attributes. Apart from the author's opinion that concepts in natural languages are formed – of course mainly unconsciously – much along this line, we have here a tool to deal with concepts and their interrelations on a computer, which may be of importance for some applications in *artificial intelligence*: automatic classification, information retrieval, data bases, expert systems, automatic theorem proving and machine translation. – A program has already been implemented. (Author)

. . . les géomètres veulent traiter géométriquement ces choses fines, et se rendent ridicules. . .

B. Pascal, Pensées, 1, Différence entre l'esprit de géométrie et l'esprit de finesse.

1. Introduction

Concepts and their handling have been pondered over for more than two thousand years, at least since Socrates; how intensively, perseveringly and portentously is shown e.g. by the "Universalienstreit" ("the great debate about universals which was to divide the schools for four centuries." (11), p.200) and its modern aftermath (19). These considerations were recently revitalized by the fact, among others, that "Artificial Intelligence" – i.e. the attempt to let a computer imitate non-numerical performances of intelligence – cannot succeed without any treatment whatsoever of concepts, cf. say (18), (15), (21) and (6), e.g. with questions of automatic classification, of data bases or expert systems, with problems of "machine learning", of automatic theorem proving or with a computer simulation of conceptual thinking in general.

Without being able to look into the historical developments, cf. e.g. (3), (2), (10), (13) and (11), this article offers a representation of concepts which enables us to work with them in a computer, of course in a coarser and simpler manner. Hereby we assume that a concept $\mathfrak{c}$ can be described precisely enough by attributes in such a way that we can always state whether an attribute, relevant for $\mathfrak{c}$, applies to $\mathfrak{c}$ or does not apply to $\mathfrak{c}$ or whether it remains open which of these two cases holds. This leads almost inevitably to a four-valued logic and to concept lattices which correspond to, indeed are even isomorphic to, those defined by R. Wille as "dichotomic" (21), 3. They often occur as building stones for more complex concept lattices (23). – In order not to frighten non-mathematicians away from this text we have tried to ban the mathematics used into part 5.

R. Wille's more general and also differently motivated approach has already led to a vast theory, cf. say (4)–(6), (14) and (21)–(24), which is by no means finished and is also eminently suited for the above mentioned purposes.

In order to distance ourselves from certain occasionally occurring conceptions let us emphasize that for us a concept – also a so-called individual concept! – is an abstract which exists independently whether or not there happens to be a word in a natural language denoting exactly this abstract.

Another introductory remark: with the representation of concepts we are concerned with, of course we are thinking primarily of uniquely definable concepts, such as in mathematics, and we do not presume to have found a tool permitting us to describe and to analyze adequately very rich – and frequently also vague – concepts, e.g. from the spheres of theology, philosophy or art.

By $\mathbb{N}$ we denote the set of natural numbers, i.e. positive integers, thus

$$\mathbb{N} = \{1, 2, 3, \ldots\},$$

and by $\mathbb{A}_m$ the set of the first m ($\in \mathbb{N}$), thus

$$\mathbb{A}_m = \{x \in \mathbb{N} | x \leq m\} \quad (m \in \mathbb{N}).$$

2. Full concept lattices

To facilitate understanding of the general developments may we begin with a simple, almost classical, instance:

Example 1:

Considering quadrangles in the Euclidean plane of our perception let a_1 denote the attribute "equal angles" and a_2 the attribute "equal sides".

These two attributes are logically independent of each other, i.e. neither the presence of a_1 implies that of a_2, nor the applying of a_2 that of a_1. Or, in other words, there are quadrangles which have equal angles but unequal sides, and such where the sides are equal but where the angles are different.

With the attributes a_1 and a_2 the concepts rectangle, square and rhombus can be easily described, e.g. by stating the pairs (g,o), (g,g) and (o,g), where a g or an o in the j–th place means that the attribute a_j is given or *o*pen (j = 1,2), respectively.

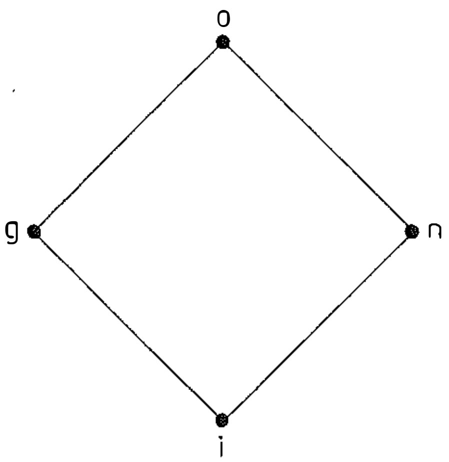
	equal angles	equal sides
rectangle	g	o
square	g	g
rhombus	o	g
proper rectangle	g	n
proper rhombus	n	g

However also concepts such as "proper rectangle" (i.e.a rectangle which is no square) or "proper rhombus" (i.e. a rhombus which is no square) can easily be reproduced in our "shorthand", e.g. by (g,n) or (n,g), respectively, where n stand for "*n*egated".

Here we shall describe no further concepts by the two attributes a_1 and a_2; this will be done below, completely and more generally, cf. e.g. ex.2.

Since when one defines a concept it seems impossible to predicate and deny one and the same attribute simultaneously[2] but as one wants to have – at least for formal reasons – a conjunction, "and"-connection, of "given" and "negated", we add to g, n and o another "truth-value" i for "*i*mpossible". – Dually to this we associate a disjunction, "or"-connection, of g and n with "open". – n lends itself as negation, denial, of g and v.v., and i as negation of o. – This suggests the formal

Definition 1: On the set T of the four *truth-values* g, n, o and i let *conjunction* ($\wedge$, and) and *disjunction* ($\vee$, or) be meet and join in the lattice $T = (T, \wedge, \vee)$ characterized by this Hasse-diagram; let the *negation* ($^-$, not) be the complement in T.



This "four-valued logic" is, incidentally, different to that developed by D. Scott (17).

For the following we assume first an arbitrary – non-empty, finite or infinite – sequence of logically independent attributes a_j (with j out of a suited index set J), the attribute sequence $A = (a_j)_{j \in J}$. In order to reach the concepts belonging to A, we first form lists, i.e. sequences, of truth-values corresponding to our ex.1, say

(g, n, o, i, g, ...), (n, g, g, o, n, ...), ...

etc. Hereby we intend to interpret a g, n, o or i in the j–th position so that the attribute a_j is given, negated, open or impossible, respectively ($j \in J$).

Since lists containing one i or several can hardly be distinguished as far as meaning is concerned we here unite the i containing lists to a concept $\mathfrak{i}$, the impossible concept, and thus come to the formal

Definition 2: Let $A = (a_j)_{j \in J}$ be an arbitrary – non-empty, at most denumerably infinite – sequence:

a) The terms a_j ($j \in J$) are called *attributes* and A is called an *attribute sequence.*

b) A *list* belonging to A is a sequence $(t_j)_{j \in J}$ with $t_j \in T$ (s. def. 1) for every $j \in J$; we denote the set of these lists by L_A.

c) Let a *concept* belonging to A be a list out of L_A which does not contain i, or the impossible concept $\mathfrak{i}$ where $\mathfrak{i}$ is an arbitrary element different to every list not containing i. The set of concepts belonging to A is denoted by C_A.

In part 5 we shall indicate how this definition can be refined. – The restriction of the index set J – essentially to $\mathbb{A}_n$ with $n \in \mathbb{N}$ or to $\mathbb{N}$ – is purely technical and can also be dropped with more than countably many attributes.

First we consider

Example 2: For an attribute sequence $A = (a_1, a_2)$ – say from ex. 1 – altogether $3^2 + 1 = 10$ concepts result.

It now appears natural to perform the operations of conjunction, $\wedge$, and disjunction, $\vee$, as fixed in def. 1 according to the components, i.e. for every attribute individually. Hereby it would make sense to require in addition that should an i appear somewhere while $\wedge$ is being carried out, one should put $\mathfrak{i}$ as the total result and besides

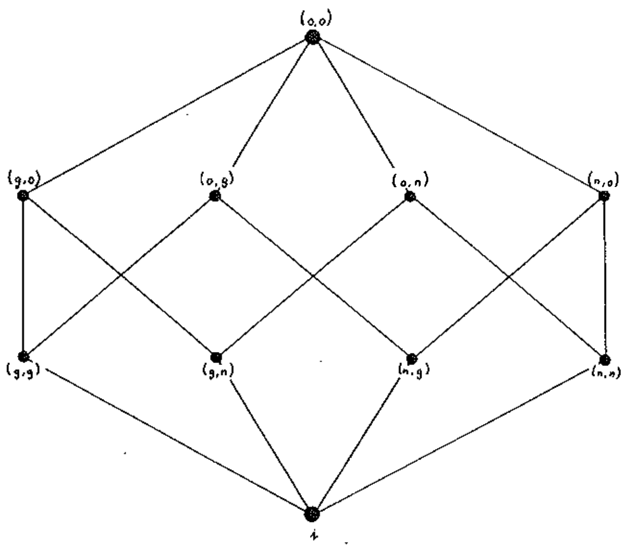
$$\mathfrak{c} \wedge \mathfrak{i} = \mathfrak{i} = \mathfrak{i} \wedge \mathfrak{c}$$

and

$$\mathfrak{c} \vee \mathfrak{i} = \mathfrak{c} = \mathfrak{i} \vee \mathfrak{c}$$

should hold for every concept $\mathfrak{c}$ out of C_A.

With these operations $\wedge$ and $\vee$ for our A with two elements the set C_A becomes a lattice $C_A = (C_A, \wedge, \vee)$ with the following Hasse-diagram



This result immediately leads to

Definition 3: For concepts $\mathfrak{c}, \mathfrak{d} \in C_A \setminus \{\mathfrak{i}\}$ of an attribute sequence $A = (a_j)_{j \in J}$, say

$$\mathfrak{c} = (c_j)_{j \in J},\ \mathfrak{d} = (d_j)_{j \in J} \text{ with } c_j, d_j \in T \setminus \{i\}\ (j \in J),$$

let $\wedge$ (*meet*, conjunction) and $\vee$ (*join*, disjunction) be defined by

$$\mathfrak{c} \wedge \mathfrak{d} = \begin{cases} (c_j \wedge d_j)_{j \in J} \\ \mathfrak{i} \end{cases} \quad \text{if} \begin{cases} c_j \wedge d_j \neq \mathfrak{i} \text{ for all } j \in J \\ \text{there is a } j \in J \text{ with } c_j \wedge d_j = \mathfrak{i} \end{cases}$$

and

$$\mathfrak{c} \vee \mathfrak{d} = (c_j \vee d_j)_{j \in J};$$

moreover, let

$$\mathfrak{c} \wedge \mathfrak{i} = \mathfrak{i} = \mathfrak{i} \wedge \mathfrak{c},$$
$$\mathfrak{c} \vee \mathfrak{i} = \mathfrak{c} = \mathfrak{i} \vee \mathfrak{c}.$$

For the structure $(C_A, \wedge, \vee)$ achieved in this way we write C_A.

The fact that a lattice thus again results also in the general case – with some here not primarily interesting algebraic properties –, is the main content of

Theorem 1: Let A be an attribute sequence with index set J and $a = |J|$. Then C_A is a complete complementary atomistic lattice of order $3^a + 1$ with $\mathfrak{i}$ as zero and $(o)_{j \in J}$ as unit; moreover, C_A possesses 2^a atoms, $a2^{a-1}$ hyperatoms and 2a antiatoms. C_A is in general, i.e. for $a > 1$, neither semimodular nor orthocomplementary.

We shall give the proof in part 5 with the tools which are made available there.

Concerning concepts $\mathfrak{c}$ and $\mathfrak{d}$ of an attribute sequence A one denotes in a natural manner $\mathfrak{c}$ as subconcept of $\mathfrak{d}$ – and correspondingly $\mathfrak{d}$ as superconcept of $\mathfrak{c}$ –, if $\mathfrak{c}$ lies under $\mathfrak{d}$ with respect to the order relation, $\leq$, induced by the lattice C_A, thus if $\mathfrak{c} \leq \mathfrak{d}$ holds. Further it appears natural to define for every concept $\mathfrak{c}$ out of C_A an opposed concept $\tilde{\mathfrak{c}}$ interchanging, roughly spoken, g with n. – These indications give rise to

Definition 4: Let $A = (a_j)_{j \in J}$ be an attribute sequence and $\mathfrak{c}, \mathfrak{d} \in C_A$:

a) C_A is called *full concept lattice* – belonging to A – and let $\leq$ be the order it induces, thus

$$\mathfrak{c} \leq \mathfrak{d} <=> \mathfrak{c} \wedge \mathfrak{d} = \mathfrak{c}.$$

b) $\mathfrak{c}$ is named *subconcept* of $\mathfrak{d}$, *and* $\mathfrak{d}$ *superconcept* of $\mathfrak{c}$, if $\mathfrak{c} \leq \mathfrak{d}$ holds.

c) Because of the completeness of C_A for every nonempty subset C of C_A there is a uniquely determined supremum, $\bigvee C$, and infimum, $\bigwedge C$; this supremum or infimum is called *smallest superconcept* (genus proximum) or *biggest subconcept* of C, respectively.

d) The concept $\tilde{\mathfrak{c}}$ is *opposed* or *contrary* to $\mathfrak{c} = (c_j)_{j \in J}$ if

$$\tilde{\mathfrak{c}} = \begin{cases} (\bar{c}_j)_{j \in J} \\ \mathfrak{i} \\ \mathfrak{o} \end{cases} \quad \text{if } \mathfrak{c} \begin{cases} \in C_A \setminus \{\mathfrak{i}, \mathfrak{o}\} \\ = \mathfrak{o} \\ = \mathfrak{i} \end{cases}$$

where $\bar{g} = n$, $\bar{n} = g$, $\bar{o} = o$ and $\mathfrak{o} = (o)_{j \in J}$.

By th. 1 and def. 4 one immediately gets

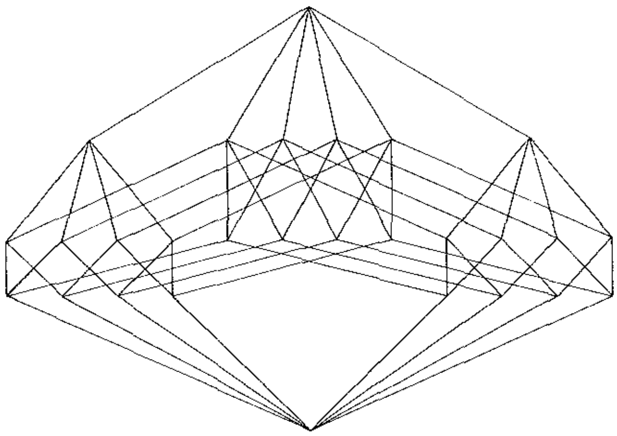
Theorem 2:

a) Exactly to the numbers b of the form $3^a + 1$ with $a \in \mathbb{N} \cup \{|\mathbb{N}|\}$ there is always – up to isomorphy – exactly one full concept lattice of order b.

b) Every full concept lattice contains – up to isomorphy – all full concept lattices of a smaller order. To be more precise: if *C* and *D* are full concept lattices of order c and d, respectively, with $c \leq d$, then there is an isomorphism from *C* into *D*.

Because of the order $3^a + 1$ of a full concept lattice the "combinatorial explosion" can also be observed here. So we give as a further illustration

Example 3: The diagram of a full concept lattice with 3 – independent – attributes, hence with $3^3 + 1 = 28$ elements (s. also (23), fig. 4 – fig. 6), results

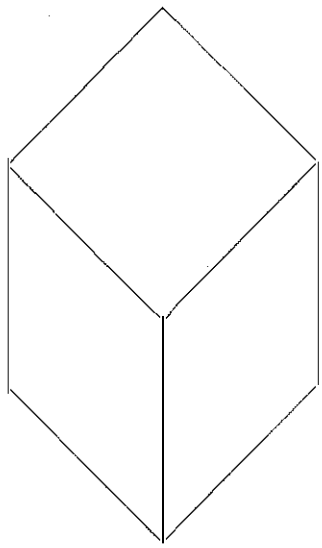


3. Reduced concept lattices

Until now we have always assumed that the single attributes of an attribute sequence under consideration were logically independent of each other. However this is in fact relatively seldom the case even in mathematical concept definitions. In most cases the applying of certain attributes implies the presence, or also the absence, of certain others. This is illustrated by

Example 4: Returning to ex. 1 we again treat quadrangles in the Euclidean plane with the attribute sequence (a_1, a_2) where now, however, a_1 no longer stands for "equal angels", but for "parallel opposite sides" and a_2 means, as before, "equal sides".

Because a rhombus is always a parallelogram the combination (n,g) turns out to be contradictory – therefore forbidden – and the combination (o,g) superfluous, since its meaning coincides with that of (g,g), just as (n,o) with (n,n). The following concepts retain their original meaning: $\mathfrak{o}$ (general quadrangle), (g,o) (parallelogram), (g,n) (proper parallelogram, i.e. parallelogram which is no rhombus), (o,n) (non-rhombus) and $\mathfrak{i}$ as the impossible concept. Thus we come to the following lattice, "emaciated" compared with that of ex. 2,



which evidently reflects reality exactly.

More generally, dependences of attributes can obviously be described in such way that, starting from a full concept lattice, one eliminates certain atoms – i.e. concepts different to i, which contain no o – and one then carries out the reduction explained in ex. 4 systematically and repeats it "towards the top" if necessary. Evidently in this way again a lattice results which we want to call *reduced concept lattice.*

This process has been well and accurately described by T. Schmottlach (16) after developping the necessary algebraic tools. However the problem of a "pleasing" characterization of the resulting lattices seems to be still unsolved.

4. Applications

First it should be emphasized that in my opinion the formation of concepts does indeed occur within the framework indicated, not only in arts and science but also in our everyday language. For in the formation of concepts it seems to be essential that, on one hand, whether or not an attribute applies can be left open, but that on the other hand one has to explicitly forbid, to negate, the applying of an attribute. We exemplify this by

Example 5: An adult who is neither divorced nor widowed can be a husband, bachelor, wife or spinster, corresponding to the atoms of ex. 2 in the given order if the first attribute is "male" and the second "married".
In English there are also words for the missing concepts in this hierarchy except for i, e.g. the "genus proximum" to "husband" and "wife" is "married"; normally, however, even in mathematics, there are no single words for all concepts of a certain attribute sequence, but nevertheless the concepts exist of course as abstracts.
According to our def. 4 the opposed concept to "husband" is "spinster" and v.v., to "bachelor" "wife" and v.v., which I think one could agree with.

Incidentally, scientific concepts seem to be formed more along the ascending and descending lines of the concept lattice in question, while everyday language appears to work more along the horizontal.

The obvious trend to increasingly "intelligent" information systems and to even more efficient expert systems is a hard challenge to artificial intelligence, especially in the field of concept analysis and processing (cf. say (15), esp. 3., p. 22–24, and (18)). Hereby one has to pay special attention to the not yet fixed or defined, i.e. to the "open" in our nomenclature, as is shown by the problem of null values in data bases, which still remains without a satisfying solution in spite of arduous attempts. It is naturally a help in information retrieval to be able to "compute" quickly sub- and superconcepts or contrary concepts and "neighbouring" or "related" concepts within the proposed lattices, e.g. with automatic library search using key words and taking into account adjacent concepts. – Machine translation from one natural language into another seems to me to be a further field for a successful application of our "semantic" efforts.

Within her "Softwarepraktikum" at the Institute for Computer Science of the Technical University Clausthal in winter 1984/85 S. Bierwirth wrote a UCSD-PASCAL-program "Begriffe" and implemented it on a SIRIUS 1 (1). The program is based on the ideas developed here; it permits one to construct concept lattices by means of attributes, to give names to the individual concepts, to compute super- and subconcepts or contrary concepts, as well as all concepts of a fixed distance to a given concept, not only for full but also for reduced concept lattices. – Meanwhile R. Wille's Darmstadt group has developed far more efficient and faster programs for different aspects of concept analysis, s. (4) or (6).

5. Mathematics

As already stated the mathematical conceptions and connections behind the developments of the former chapters, esp. part 2, will be described in this section as shortly as possible.

Let P(S) stand for the power set of a set S and let

$$1_S \leftrightharpoons \{(s,s) \mid s \in S\}.$$

As far as lattice theoretical nomenclature and notation are concerned we refer largely to (7).

A simple generalization of the concept of an ideal in a lattice is needed: let us remember here (cf. say (8), I. 7), that every *ideal* I of a semigroup $S = (S, \cdot)$, in short $I \trianglelefteq S$, i.e. $I \in P(S) \setminus \{\emptyset\}$ with SI, IS $\subseteq$ I, induces the *Rees congruence*

$$\equiv \leftrightharpoons (I \times I) \cup 1_S$$

and thus the *Rees quotient* $S/I \leftrightharpoons S/\equiv$ which still makes sense for $I = \emptyset$ too: $S/\emptyset \simeq S$. – We give

Definition 5: Let $L = (L, \wedge, \vee)$ be a lattice and $K \subseteq L$: if $K = \emptyset$ or $K \trianglelefteq (L, \wedge)$, then K is called *contractable* (with respect to L).

For every lattice $L = (L, \wedge, \vee)$, trivially, L itself is contractable and also $\emptyset$, but every ideal of L too; further examples will be mentioned immediately after def. 6. – The following statement justifies our nomenclature.

Lemma: In a lattice $(L, \wedge, \vee)$ let K be contractable and let the class belonging to $x \in K$ of the equivalence relation $(K \times K) \cup 1_L$ be denoted by $\hat{x}$. Furthermore let $(C, \wedge) \leftrightharpoons (L, \wedge)/K$ and

$$\hat{x} \vee K \rightleftarrows \hat{x} \rightleftarrows K \vee \hat{x} \quad (x \in L)$$

if $K \neq \emptyset$, moreover let

$$\forall\, x, y \in L \setminus K: \quad \hat{x} \vee \hat{y} \leftrightharpoons \widehat{x \vee y}.$$

Then $C = (C, \wedge, \vee)$ is a lattice which has K as zero if $K \neq \emptyset$.

Proof: These different assertions can easily be verified by a simple computation taking several special cases into account.

This lemma gives rise to

Definition 6: Let the lattice C constructed according to the lemma be called *contract* of L with respect to K, in short $C = C_K(\dot{L})$.

For every lattice $L = (L, \wedge, \vee)$ holds trivially

$$C_L(L) \in \mathfrak{C}_1; \tag{1}$$

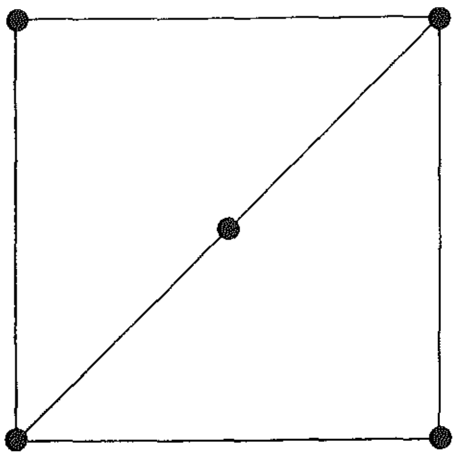
$C_L(L)$ is thus a trivial lattice, i.e. a singleton. (Let $\mathfrak{C}_n$ in general denote the isomorphy class of a chain with n elements (7), p. 16.) In addition is

$$C_{\emptyset}(L) \simeq L \tag{2}$$

and if L has a zero z also $C_{\{z\}}(L) \simeq L$. Further

Example 6:

a) With $K = \{\emptyset, \{1\}, \{2\}, \{3\}\}$ is $C_k(P(A_3), \cap, \cup)$ out of $\mathfrak{M}_3$ where $\mathfrak{M}_3$ denotes the isomorphy class of the diamond, hence of the lattice of this diagram (7), p. 59.



b) For lattices $L_j = (L_j, \wedge, \vee)$ with zero $z_j\ (\in L_j)$ for $j \in J$ let P be the direct product of the L_j, say $P = (P, \wedge, \vee) = \bigotimes_{j \in J} L_j$, and Z the set of elements of P, which contain a zero, thus

$$Z \leftrightharpoons \{(x_j)_{j \in J} \in P \mid \exists\, k \in J: x_k = z_k\};$$

then $Z \trianglelefteq (P, \wedge)$ and $C_z(P)$ is a well-defined contract.

c) For $K = \{\emptyset, \{1\}\}$ one has $C_K(P(A_2), \cap, \cup) \in \mathfrak{C}_3$.

This last example shows that the complementariness of a lattice can get lost[3], just as the distributivity – as evident by ex. 6a –; even modularity and semimodularity do not remain (s. e.g. 2., th. 1). If, however, a lattice is complete or algebraic so is every one of its contracts.

By means of the now available tools we have to show th. 1 of 2.:

Proof of th. 1: Using the notations of def. 1 and def. 2b we form the lattice of lists for A

$$L_A \leftrightharpoons (L_A, \wedge, \vee) \leftrightharpoons \bigotimes_{j \in J} T,$$

and the set I of the i containing lists, thus

$$I \leftrightharpoons \{(t_j)_{j \in J} \in L_A \mid \exists\, k \in J: t_k = i\}.$$

By ex. 6b

$$K = (K, \wedge, \vee) = C_I(L_A)$$

is a contract. From def. 3 it follows that $K \simeq C_A$ and therefore it suffices to prove the assertion for K:

L_A is as a direct power of a complete Boolean lattice T also a complete Boolean lattice, and hence K, as a contract of L_A, is complete according to the above remark.

With the notations of def. 4d

$$\forall\, \mathfrak{c} \in K: \ \mathfrak{c} \wedge \tilde{\mathfrak{c}} = \mathfrak{i},\ \mathfrak{c} \vee \tilde{\mathfrak{c}} = \mathfrak{o}$$

holds, which can be immediately verified by distinction of the various cases; this shows K as a lattice.

Let J be the index set belonging to A and U the set of atoms in K. Since it is evident that

$$U = \{\widehat{(t_j)_{j \in J}} \in K \mid \ \forall\, j \in J: t_j \in \{g, n\}\},$$

one has $|U| = 2^a$ and K is atomistic, i.e. every nonzero element of K is a join of atoms (s. (7), p. 179). – The other statements about cardinalities are just as easily verified.

Semimodularity (s. (7), p. 172) means the validity of

$$\forall\, \mathfrak{c}, \mathfrak{d}, \mathfrak{e} \in K: \ \mathfrak{c} \mathrel{-\!\!<} \mathfrak{d} \ \Rightarrow \ \mathfrak{i} \vee \mathfrak{e} \mathrel{=\!\!<} \mathfrak{d} \vee \mathfrak{e},$$

where "$x \mathrel{-\!\!<} y$" is the abbreviation for "x is the lower neighbour of y". As is obvious from the definition of K we have

$$\mathfrak{i} \mathrel{-\!\!<} \mathfrak{g} \leftrightharpoons (g)_{j \in J},$$

but, since

$$(n)_{j \in J} \rightleftarrows \mathfrak{n} = \mathfrak{i} \vee \mathfrak{n}$$

and $\mathfrak{g} \vee \mathfrak{n} = \mathfrak{o}$, not

$$\mathfrak{i} \vee \mathfrak{n} \mathrel{=\!\!<} \mathfrak{g} \vee \mathfrak{n}$$

for $a > 1$, whereby K is shown to be not semimodular in this case.

We assume that K is orthocomplementary, i.e. to every $\mathfrak{c} \in K$ there is a complement $\mathfrak{c}'\ (\in K)$ with $\mathfrak{c}'' = \mathfrak{c}$ and

$$\mathfrak{c} \leq \mathfrak{d} \ \Rightarrow \ \mathfrak{d}' \leq \mathfrak{c}' \quad (\mathfrak{c}, \mathfrak{d} \in K). \tag{3}$$

$\mathfrak{n}$ is the only possible complement for $\mathfrak{g}$ and v.v.; for $\widehat{(o, g, \ldots)}'$ one can choose between $\widehat{(o, n, \ldots)}$, $\widehat{(g, n, \ldots)}$ and n. Because $\mathfrak{g} \leq \widehat{(o, g, \ldots)}$ and (3) one has $\widehat{(o, g, \ldots)}' = \mathfrak{n}$ in contradiction to

$$\widehat{(o, g, \ldots)} = \widehat{(o, g, \ldots)}'' = \mathfrak{n}' = \mathfrak{g}.$$

Thus K is proven to be non-orthocomplementary for $a > 1$.

In the descriptions and analyses so far according to def. 2c we have for simplicity's sake always identified a concept with a list or with the impossible concept. In fact, however, the lists do not depend on the attribute sequence A itself, but only on the corresponding index set J. When interpreting the contents, however, one needs the attributes themselves. In order to remedy this deficiency it is enough to consider the pairs (c, A) with $c \in C_A$ as "refined concepts". Of course our structural statements remain essentially unchanged since there is a bijective correspondence between both definitions of concepts as soon as we have fixed a certain attribute sequence.

We have here used attribute sequences instead of attribute sets for purely technical reasons and this does not mean any restriction because every set can be well-ordered; on the contrary, there is a certain advantage in using sequences since equal elements may occur as terms of a sequence, a situation which, where sets are concerned, can be mastered by moving from the attributes to possibly different names.

Notes

1 This is basicly an English version of the article (12); editors and author are greatly indebted to the editors of the above mentioned volume and to the BI-Wissenschaftsverlag for their kind permission to publish. – The author sincerely thanks B. Ganter, I. Kupka, G. Pickert and R. Wille for helpful and stimulating discussions, Miss S. Bierwirth (now Mrs. Behnke) for good programming and Miss M. Söding for careful proofreading.

2 We omit here the Cusanic "coincidentia oppositorum", of importance in the history of philosophy, cf. e.g. [9] or [20], p. 250.

3 This important hint – and hence the correction of a former mistake – was given by T. Schmottlach (16), to whom the author is greatly indebted.

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FRG: Call for Papers 12th Annual Conference

From 17-19 March 1988, the German Society for Classification will hold its 12th Annual Meeting on the topic "Classification and Order" at the Technical University of Darmstadt. There will be plenary lectures and Workshops. Papers are invited for the latter on the following topics:
1) Conceptual Order, 2) Order in Languages, 3) Library Classification, 4) Information Retrieval and Databases, 5) Commodity Classification and Product Description, 6) Decision Supporting Systems, 7) Recognition of Structures in Data Analysis and Statistics, 8) Numerical Classification, 9) Order Structures in the Natural Sciences.
The plenary lectures will be delivered by H.H.BOCK, Aachen, (Statistics and Data Analysis); W.GAUL, Karlsruhe, (Decision Theory and Operations Research); H.GOEBL, Salzburg, (Dialect Research); E.HOLENSTEIN, Bochum, (Philosophy of Language); R.FUGMANN, Idstein, (Order and Information); G.LUSTIG, Darmstadt, (Information Retrieval); I.RIVAL, Ottawa, (Mathematical Order Theory), F.WINGERT, Münster, (Medical informatics and biomathematics). The deadline for abstracts was set for Nov.15, 1987.
Registration, including the conference proceedings: DM 50.- for members and DM 100.- for non-members; students have free entrance to the lectures. The program will be available at the beginning of February 1988. For further information or registration contact: Prof.Dr. R.Wille, FB Mathematik, Technische Hochschule, D-6100 Darmstadt.